



**PAMIBIA UNIVERSITY**  
OF SCIENCE AND TECHNOLOGY

**SCHOOL OF HEALTH AND APPLIED SCIENCES**

DEPARTMENT OF MATHEMATICS AND STATISTICS

<b>QUALIFICATION :</b> Bachelor of Science in Applied Mathematics and Statistics	
<b>QUALIFICATION CODE:</b> 35BAMS	<b>LEVEL:</b> 5
<b>COURSE:</b> PROBABILITY THEORY 1	<b>COURSE CODE:</b> PBT501S
<b>DATE:</b> JUNE 2019	
<b>DURATION:</b> 3 HOURS	<b>MARKS:</b> 100

<b>FIRST OPPORTUNITY EXAMINATION QUESTION PAPER</b>	
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<b>MODERATOR:</b>	Mr. A. Roux

**THIS QUESTION PAPER CONSISTS OF 5 PAGES**  
(Including this front page)

**INSTRUCTIONS**

1. Answer ALL the questions.
2. Write clearly and neatly.
3. Number the answers clearly.

**PERMISSIBLE MATERIALS**

Calculator.

**ATTACHMENTS**

Statistical tables (Z-TABLE).

**Question 1 [25 marks]**

- 1.1. Define the following terminologies as they are applied in probability theory.
- 1.1.1. A set [2]
  - 1.1.2. A random experiment [2]
  - 1.1.3. A sample space [2]
  - 1.1.4. An event [2]
  - 1.1.5. A partition of a sample space [3]
- 1.2. For any arbitrary events A and B in the sample space  $S$ , show that  
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
 [6]
- 1.3. Let  $\{B_1, B_2, \dots, B_k\}$  be a partition of  $S$  such that  $P(B_i) > 0$ , for  $i = 1, 2, \dots, k$ . Show that  
$$P(A) = \sum_{i=1}^k P(A|B_i)P(B_i)$$
 for any event A in the sample space  $S$ . [8]

**Question 2 [25 marks]**

- 2.1. Consider the experiment of tossing a fair coin three times.
- 2.1.1. Develop a tree diagram for the experiment. [2]
  - 2.1.2. List the experimental outcomes. [4]
  - 2.1.3. What is the probability for each experimental outcome? [2]
- 2.2. A class of 15 students is to be split into three tutorial groups of sizes 5, 4, and 6 respectively.
- 2.2.1. In how many ways can this be done? [4]
  - 2.2.2. There are two brothers in the class. What is the probability that they are in the same probability class? [4]

- 2.3. The following table shows household income by educational level of the head of household (Statistical Abstract of the United States, 2008).

Educational level	Household income in US\$ 1000					Tot
	under 25	25-49.9	50-74.9	75-99.9	100 or more	
Not H.S graduate	4207	3459	1389	539	367	9961
H.S graduate	4917	6850	5027	2637	2668	22099
Some college	2807	5258	4678	3250	4074	20067
Bachelor's Degree	885	2094	2848	2581	5379	13787
Beyond Bach.Degree	290	829	1274	1241	4188	7822
<b>Tot</b>	<b>13106</b>	<b>18490</b>	<b>15216</b>	<b>10248</b>	<b>16676</b>	<b>73736</b>

- 2.3.1 What is the probability of a household headed by someone with educational level beyond a bachelor's degree or by someone earning between \$50000 and \$74900?[2]
- 2.3.2 What is the probability of a household headed by someone with a bachelor's degree earning \$100,000 or more? [2]
- 2.3.3 If a household with income under \$25000 is selected at random, what is the probability that it is headed by someone with a bachelor's degree? [5]

### Question 3 [25 marks]

- 3.1. Suppose that four inspectors at a film factory are supposed to stamp the expiration date on each package of film at the end of the assembly line. John, who stamps 20% of the packages, fails to stamp the expiration date once in every 200 packages; Tom, who stamps 60% of the packages, fails to stamp the expiration date once in every 100 packages; Jeff, who stamps 15% of the packages, fails to stamp the expiration date once in every 90 packages; and Pat, who stamps 5% of the packages, fails to stamp the expiration date once in every 200 packages. If a customer complains that her package does not show the expiration date, what is the probability that it was inspected by John? [4]
- 3.2. The minister of environmental affairs has to decide on a fishing quota for forthcoming season. Currently, the biomass of fish is estimated to be 20 m tons. The fish may have a good breeding season with probability 0.3 and produce 10 m tons of young fish, or have bad breeding season and produce only 1 m ton. A so-called warm water event may occur with probability 0.1, and kill 15 tons of fish, otherwise 1 m ton of fish die.

3.2.1 Find the value of  $X$ , the biomass of fish before setting the quota, can take on. [2]

3.2.2 Assume that all events are independent; find the probability mass function of  $X$ . [3]

3.2.3 Find the mean of  $g(x) = x^2 - 1$  [3]

3.3. Let  $X$  be a random variable with the probability mass function

$$p(x) = \begin{cases} \binom{5}{x} 0.4^x 0.6^{5-x}, & \text{for } x = 0, 1, 2, 3, 4, 5 \\ 0, & \text{elsewhere} \end{cases}$$

3.3.1 Find,  $F(x)$ , the distribution function of the random variable  $X$  [6]

3.3.2 Find the median of  $X$  [2]

3.4. A random variable  $X$  has a mean  $\mu = 10$  and a variance  $\sigma^2 = 4$ . Use Chebyshev's theorem to estimate  $P[|X - 10| < 3]$ . [5]

**Question 4 [25 marks]**

4.1. A shoe store's records show that thirty percent of his customers purchase shoes via the use of a credit card. A couple of mornings ago, twenty customers purchased shoes from the store.

4.1.1 What is the probability that at least 3 customers but not more than 6 used a credit card? [5]

4.1.2 What is the expected number of customers using a credit card? [2]

4.1.3. What is the variance of the number of customers using a credit card in that shoe store? [2]

4.2 The number of bank robberies that occur in large north American city is Poisson distributed with mean of 1.8 per day.

4.2.1 Find the probability that between 10 and 12 (both values inclusive) robberies occur during a 5-day period? [4]

4.2.2 What is the expected number of bank robberies during the 5-day period? [2]

4.2.3 What is the standard deviation number of bank robberies during the 5-day period [2]

4.3. A soft-drink machine is regulated so that it discharges an average of 200 milliliters per cup. If the amount of drink is normally distributed with a standard deviation equal to 15 milliliters,

4.3.1. what fraction of the cups will contain more than 224 milliliters? [3]

4.3.2. below what value do we get the smallest 25% of the drinks? [5]

END OF EXAM PAPER

## Standard Normal Probabilities

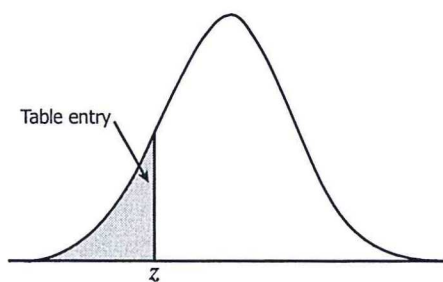
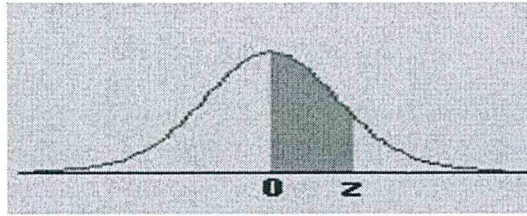


Table entry for  $z$  is the area under the standard normal curve to the left of  $z$ .

$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



# The Standard Normal Distribution



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990